Dynamic ice fronts: Implementing an empirical ice-shelf calving law.

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Why Look at Calving?

- Calving rate controls ice-shelf length
- Longer shelves will buttress outflow of grounded ice more
  - More lateral drag
  - Greater likelihood for local grounding (ice rises)
- Calving impacts the ice-sheet mass balance and thus sea-level rise

Empirical Calving "Law"?:

- Considering cold, floating termini;
- Looking for the zeroth-order relationship from velocity data
- Hypothesis: the tendency for ice shelves to fall apart (the near-front spreading rate) controls the rate at which they fall apart (the calving rate).

$$c \propto {u_x}^m$$

Width and thickness? $c \propto (HWu_x)^m$

Procedure:

- Measure long. stretching rate about one iceberg-width from the front, especially near center-line of shelf
- Measure “calving rate” (assume s.s. - not so crazy…);
- Plot up the results; do they match the hypothesis?

Calving Law

Whole data set. Positive slope is dominated by Jakobshavn (shown for three different times; J).

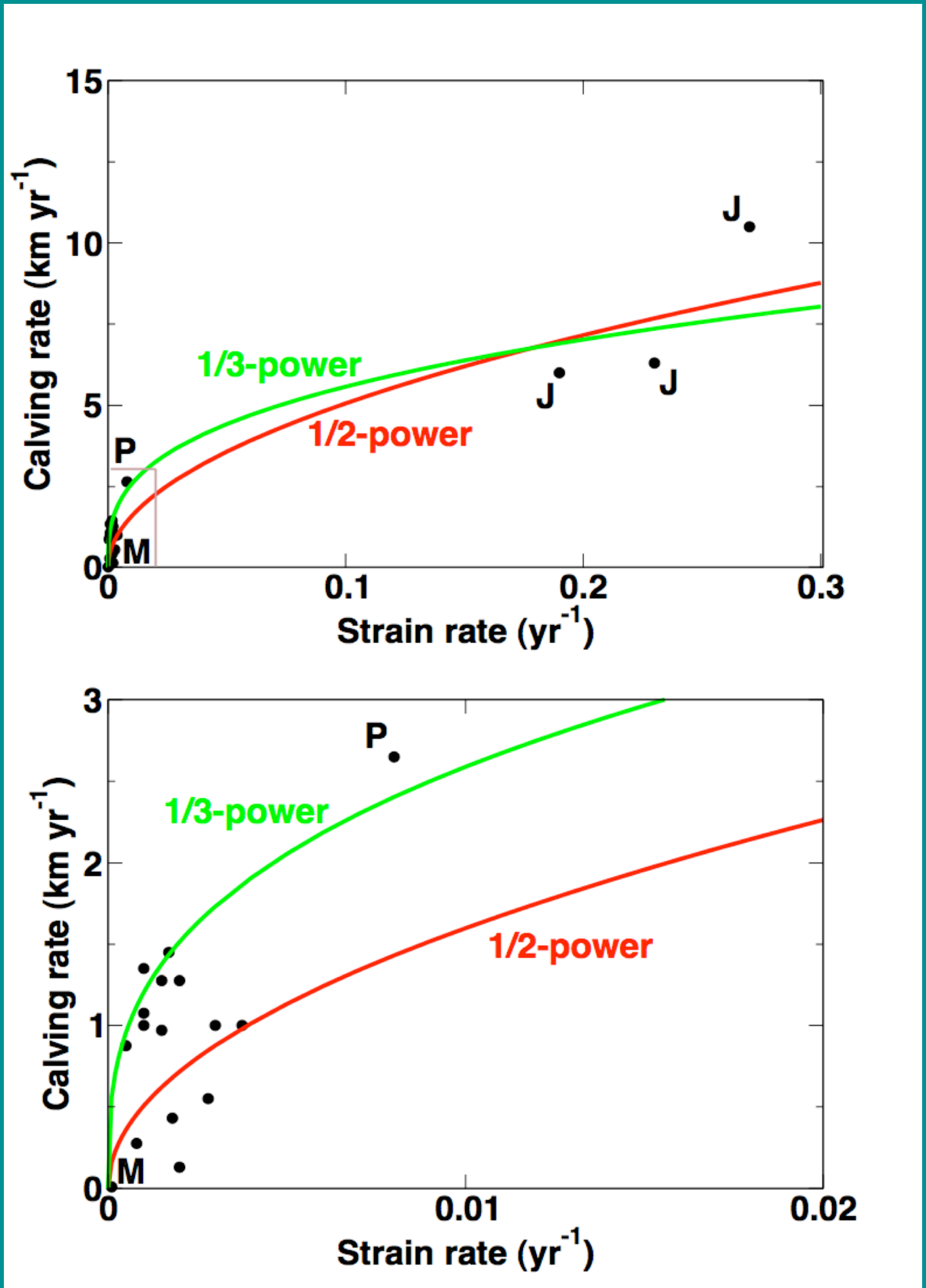
The square-root relation is consistent with fits to various subsets of the data.

Cube root works too.

Plotted line is: $c=1.6x10^{4}*u_x^{1/2}$

Explain approx. 90% of the variance

Blow-up of low-strain-rate data. Pine Island (P) and McMurdo (M) dominate. Omitting them leaves a positive-slope relation (noisy, w/ lower confidence).



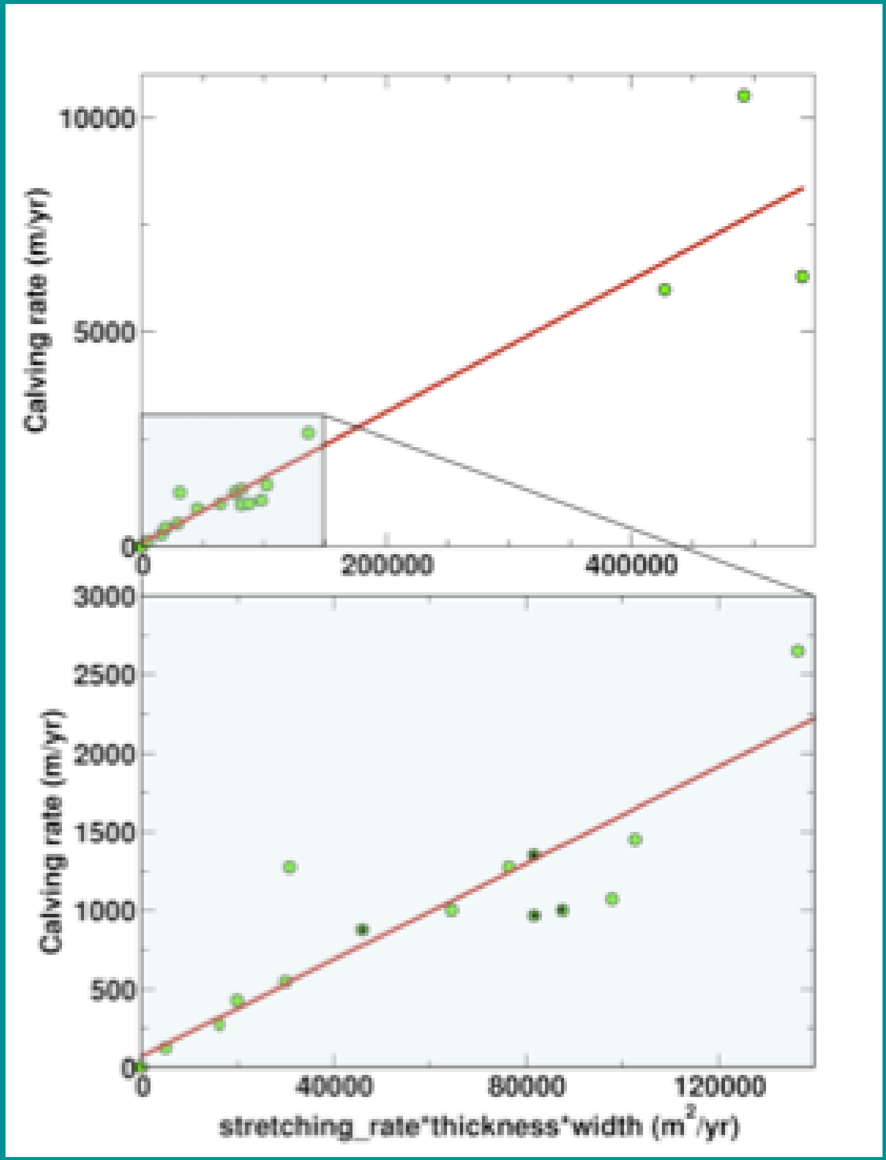
Calving Law - Including Thickness and Width

Intuition and data suggest that thicker and wider ice fronts experience faster calving

Best fit curve is: $c=0.022(Hwu_x)^{0.975}$

Plotted is: $c=70\ \mathrm{m/yr} + 0.015\ Hwu_x$

Both explain 89% of the variance



Can we use the law?

- Limitations of the law
  - empirical correlation (inspired by phys. intuition)
  - noisy
  - essentially 1-d
  - continuous, not episodic
- But say it’s of heuristic value…

Question: What might the <u>dynamic</u> consequences be?

Numerical Experiments

- Implement the calving law in a simplified model of an ice shelf
- Allow the ice front to migrate
- Is there a equilibrium ice front position?
- Is this equilibrium stable or unstable?

Model in brief

1-d, strait-sided (for now), w/ a stretching long. coordinate

$$\eta \equiv \frac{x}{x_{if}(t)}$$

Ice-front balance:

$$\partial_t x_{if} = u_{if} - c = u_{if} - A_c \left(\partial_x u|_{x_{if}} \right)^{\frac{1}{2}}$$

Mass-balance or thickness-evolution equation:
-mapped from t,x to t,η space
-neglects accumulation/ablation (for now)
-bc: const. inlet thickness

$$\partial_t h = \eta \partial_t x_{if} \frac{1}{x_{if}} \partial_\eta h - \frac{1}{x_{if}} \partial_\eta (uh), \quad 0 \le \eta \le 1$$

Stress-equilibrium equation:
-depth and width-integrated MacAyeal/Morland eqn
-lateral friction treated as boundary-layer phenom.
-bc's: ice front stretching condition, const. inlet velocity

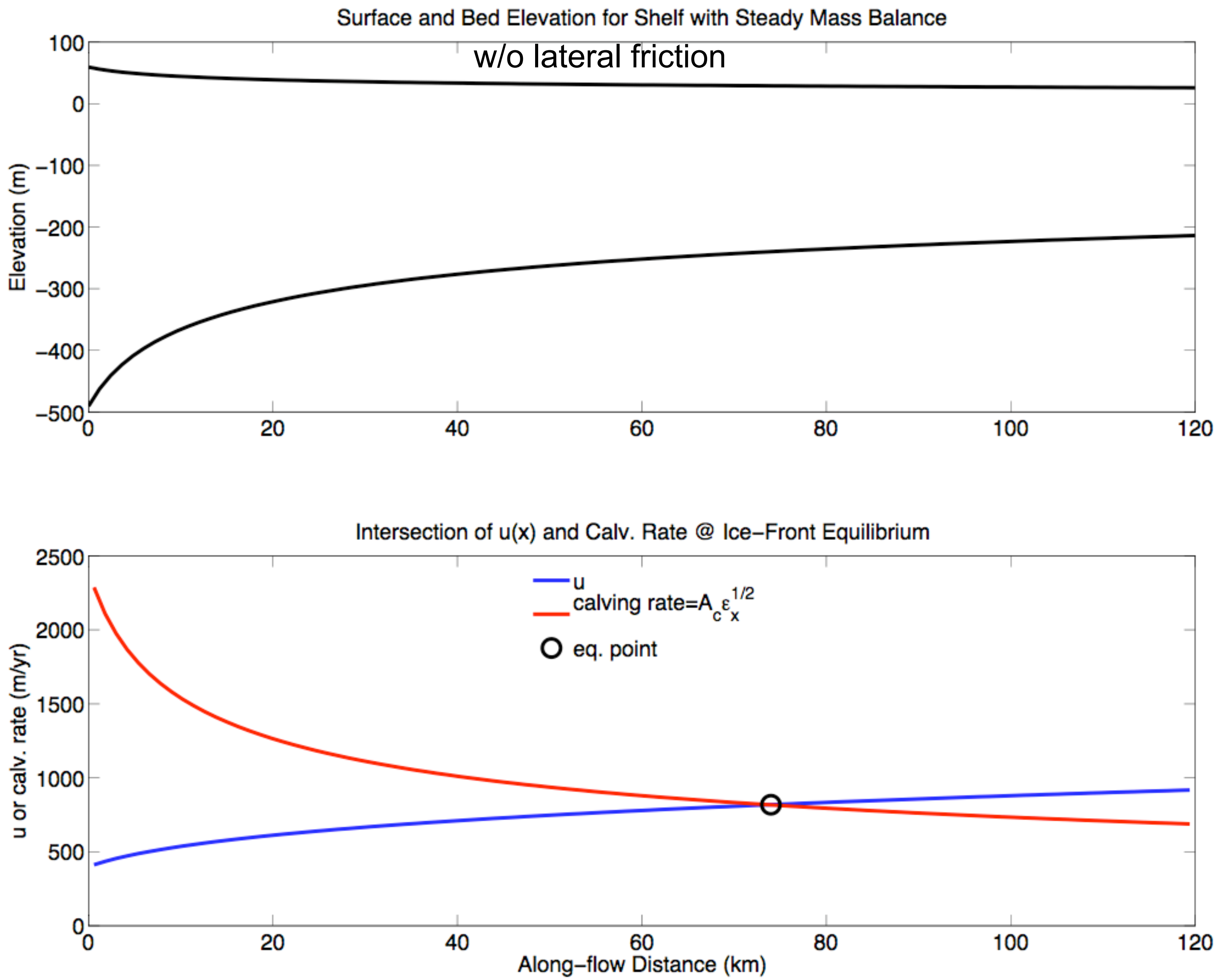
$$\frac{1}{x_{if}} \partial_\eta \left(4h\nu \frac{1}{x_{if}} \partial_\eta u - \frac{\rho_i g}{2} h^2 \right) = -\frac{\rho_i}{\rho_{sw}} \rho_i g h \frac{1}{x_{if}} \partial_\eta h + \frac{h}{L_y} \gamma_s(u)\, u, \quad \gamma_s \equiv B_s |u|^{\frac{1-n}{n}}$$

Parameter Values

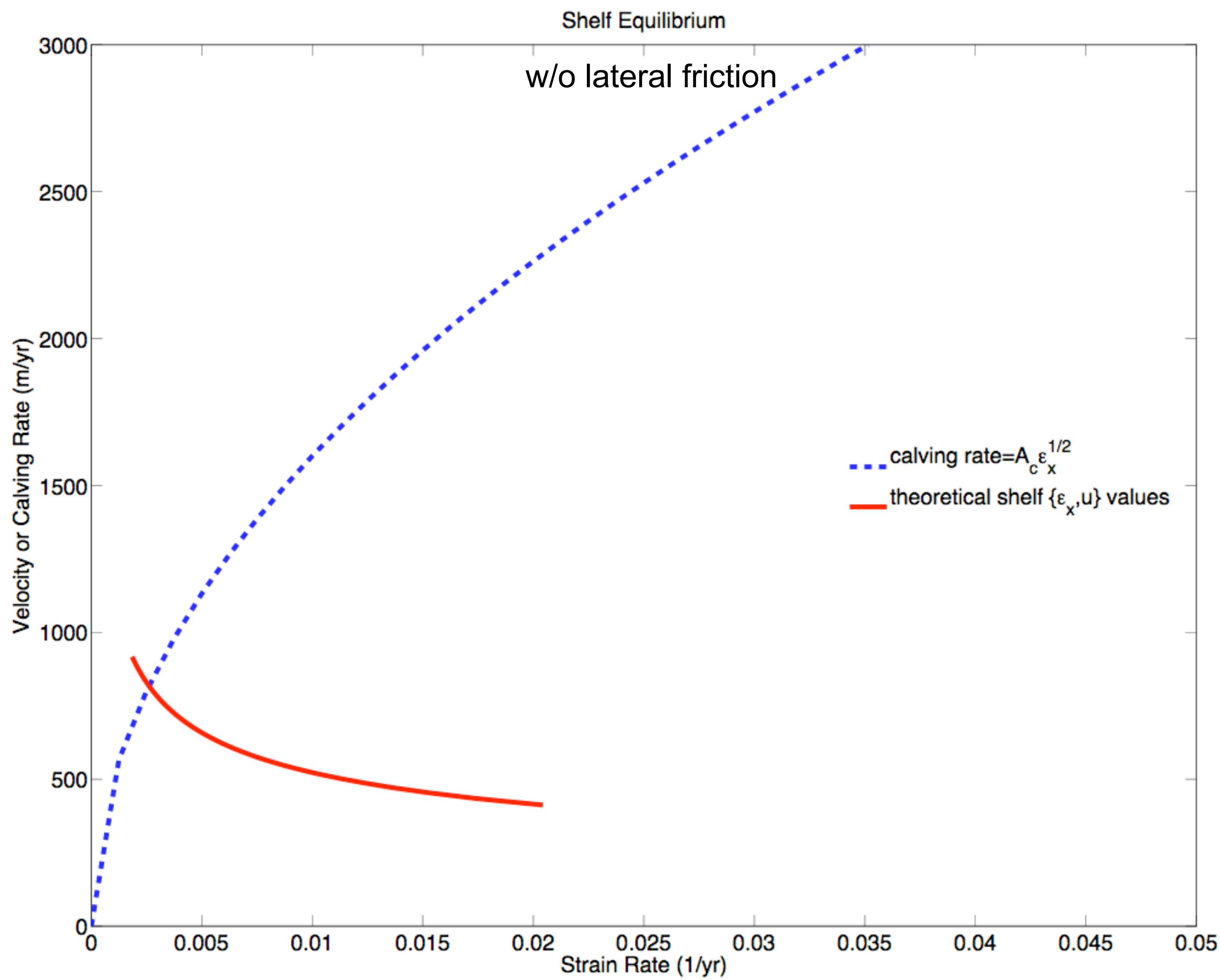
parameter	value (units)
A_c	1.6×10^4 (m $s^{-1/2}$)
g	9.81 (m s^{-2})
ρ_i	917 (kg m^{-3})
ρ_{sw}	1028 (kg m^{-3})
B_i	1.5×10^8 (Pa $s^{\frac{1}{n}}$)
n	3
h_0	550 (m)
u_0	400 (m yr^{-1})
L_y	30 (km)

Finding Equilibrium

- For what ice front position is the system at equilibrium?
  - Steady thickness (mass-balance or thickness-evolution eqn)
  - Steady ice front balance
- Straight forward procedure:
  - Hold ice front at a chosen value
  - Let the mass-balance eqn. come to eq.
  - What is the ice-front balance?
  - Change ice front pos. accordingly and iterate
- Easier for shelves w/o lateral friction
  - Plot u_x vs u for a steady (and analytic) profile and see where it crosses the calving-law curve.

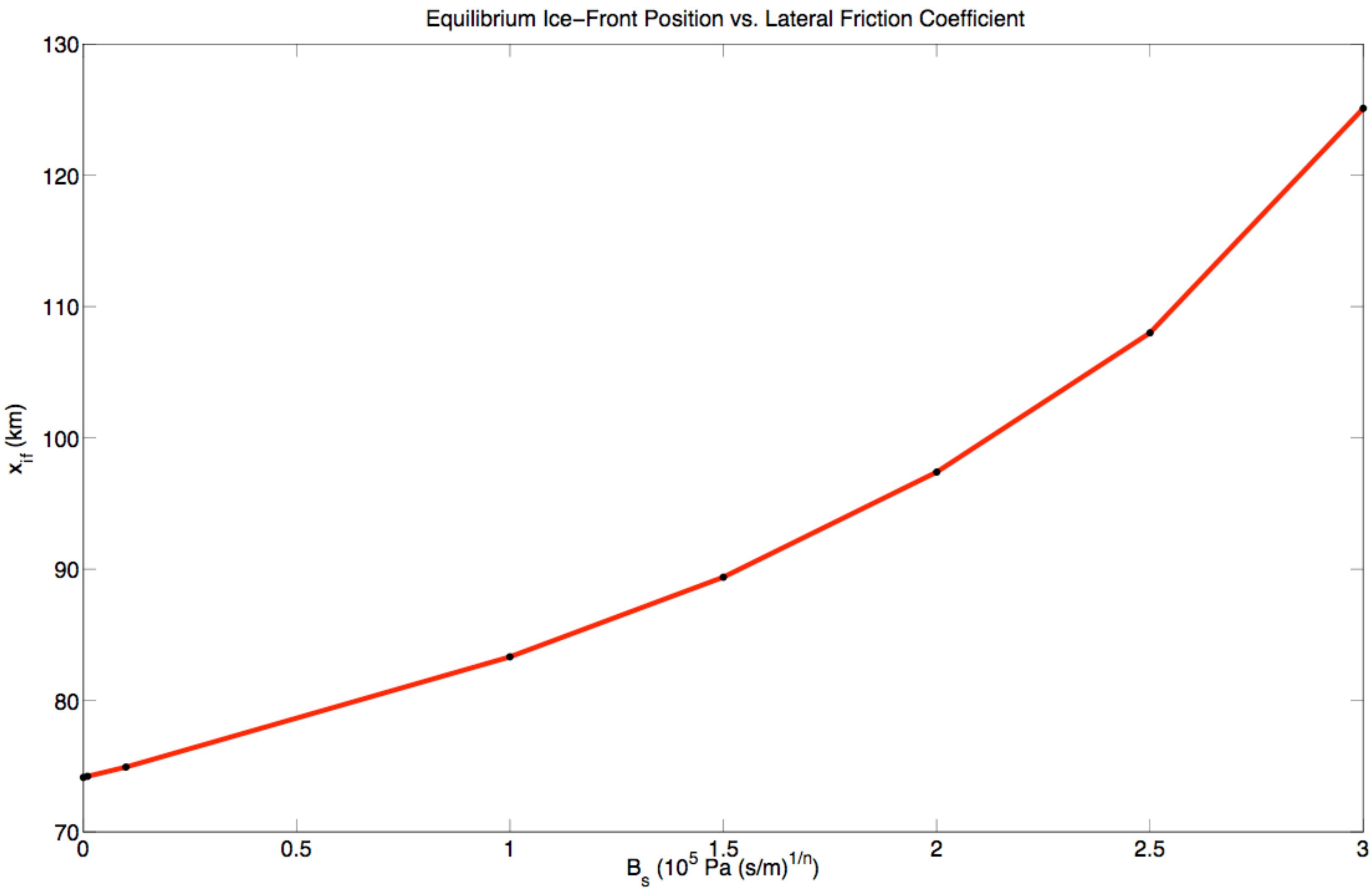


Surface and Bed Elevation for Shelf with Steady Mass Balance
w/o lateral friction
Elevation (m)
100
0
−100
−200
−300
−400
−500
0
20
40
60
80
100
120
Intersection of u(x) and Calv. Rate @ Ice−Front Equilibrium
u
calving rate=$A_c \varepsilon_x^{1/2}$
eq. point
u or calv. rate (m/yr)
2500
2000
1500
1000
500
0
Along−flow Distance (km)



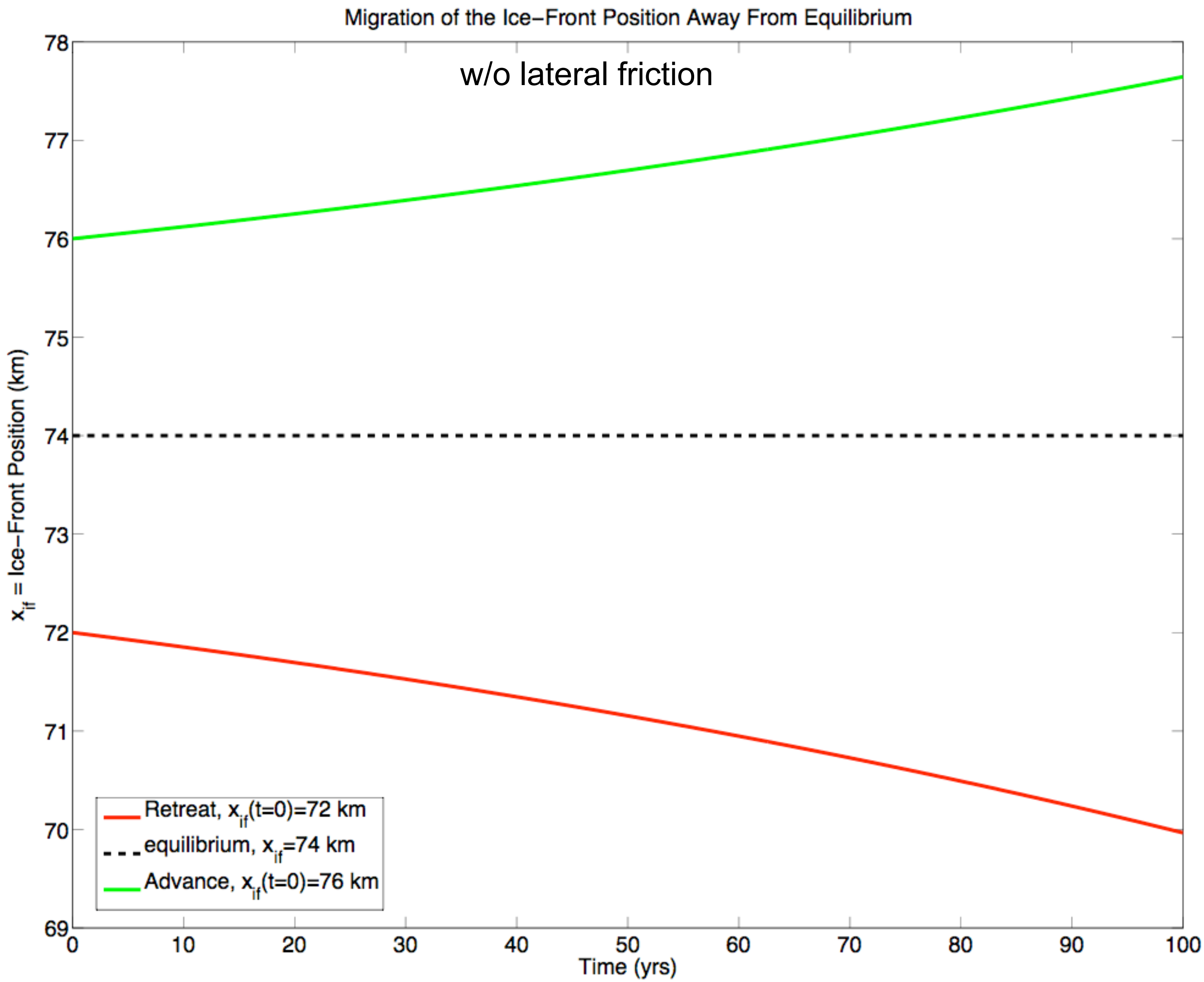
Shelf Equilibrium
w/o lateral friction
Velocity or Calving Rate (m/yr)
Strain Rate (1/yr)
3000
2500
2000
1500
1000
500
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
calving rate=$A_c \varepsilon_x^{1/2}$
theoretical shelf $\{\varepsilon_x, u\}$ values

Equilibrium Lengthens w/ Lateral Friction



Stability

- Found an ice-front position where transient terms go to zero (equilibrium) w/ and w/o lateral friction
- Question: Is that position stable?
  - Perturb the ice front position from this equilibrium value and see how the system evolves
  - Return to equilibrium position (stable) or no (unstable)

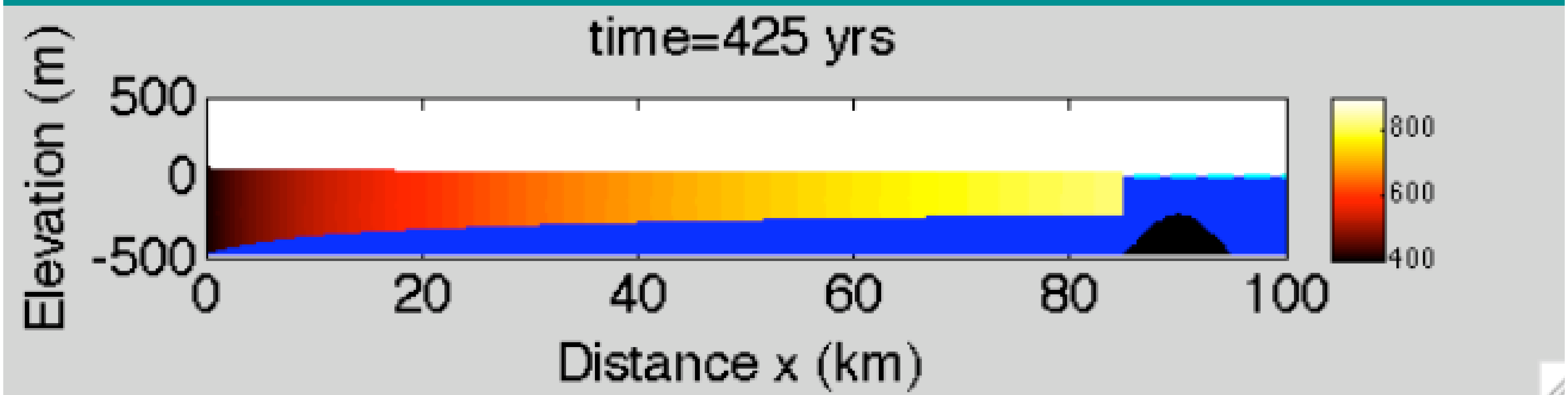


Migration of the Ice−Front Position Away From Equilibrium
w/o lateral friction
x_{if} = Ice−Front Position (km)
Time (yrs)
78
77
76
75
74
73
72
71
70
69
0
10
20
30
40
50
60
70
80
90
100
Retreat, x_{if}(t=0)=72 km
equilibrium, x_{if}=74 km
Advance, x_{if}(t=0)=76 km

Results

- The equilibrium ice-front position is unstable for $c \propto u_x^{1/2}$
- This is also true when lateral friction is included - surprising?
  - Regardless of lateral friction, a retreating ice front is thicker (--> more strain-rate) and slower
- Instability remains w/ thickness and width-dependent calving law Given the apparent quasi-steady positions of real shelves, what's wrong?
  - Law?
  - Implementation?
  - Scenarios? <-- no variable width, no local grounding

Including local grounding



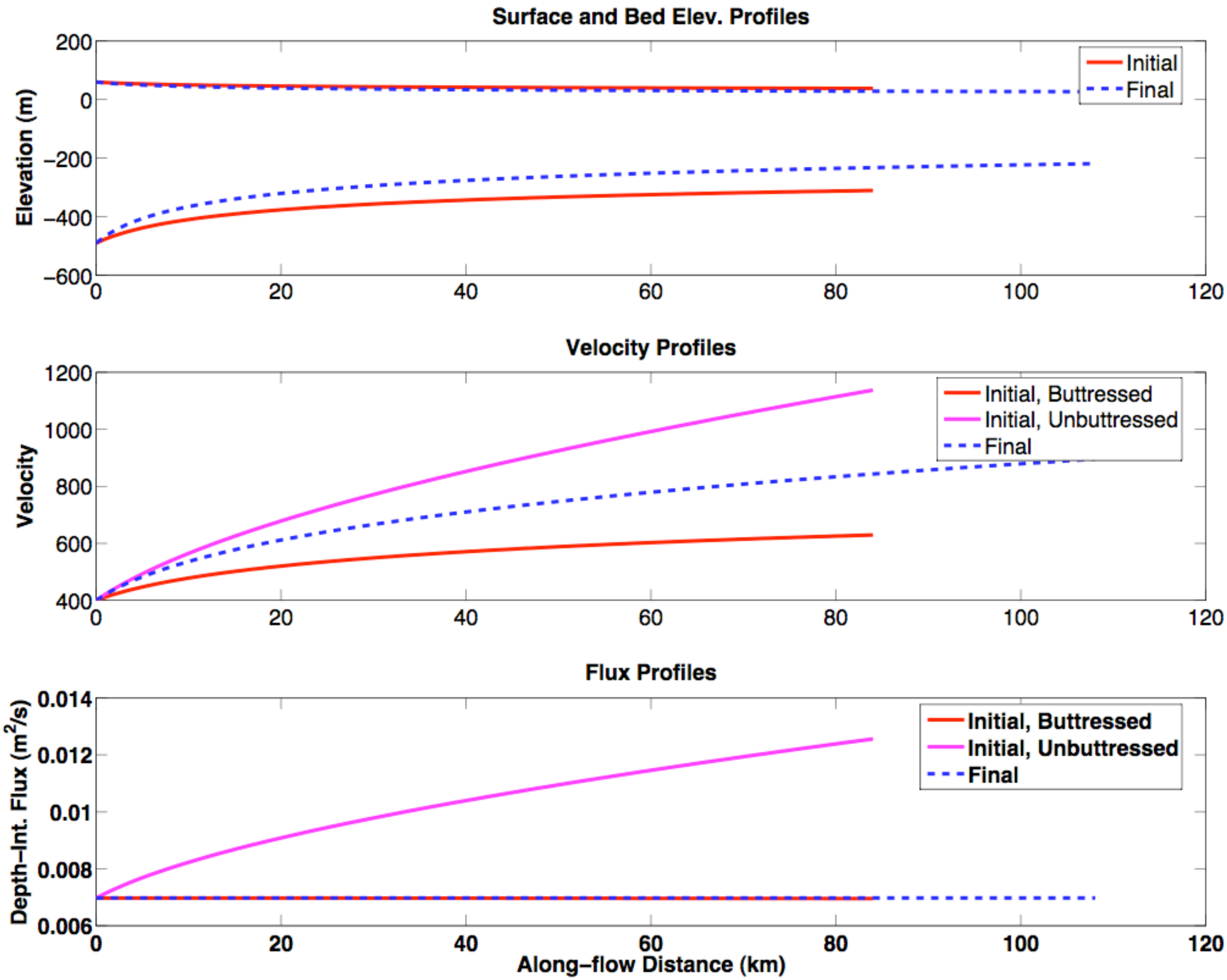
Concluding Questions

- Will along-flow width variation introduce stability?
- How do we impelment this law in a 2-d or 3-d model?
- How easy is it to employ a fixed mesh in 2-d or 3-d calving scenarios? --> <u>adaptive meshing</u>?

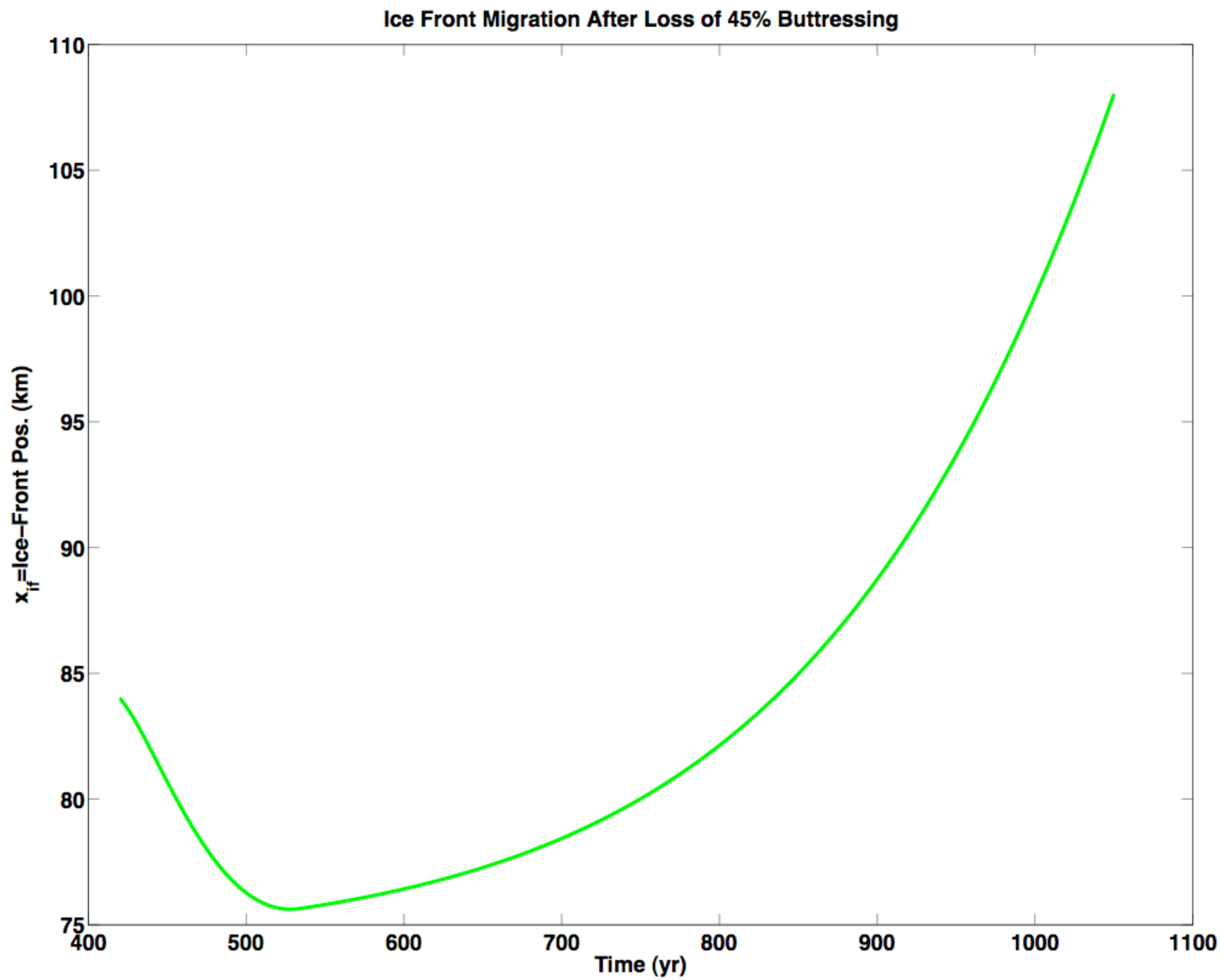

Thanks for your attention.

Working on it

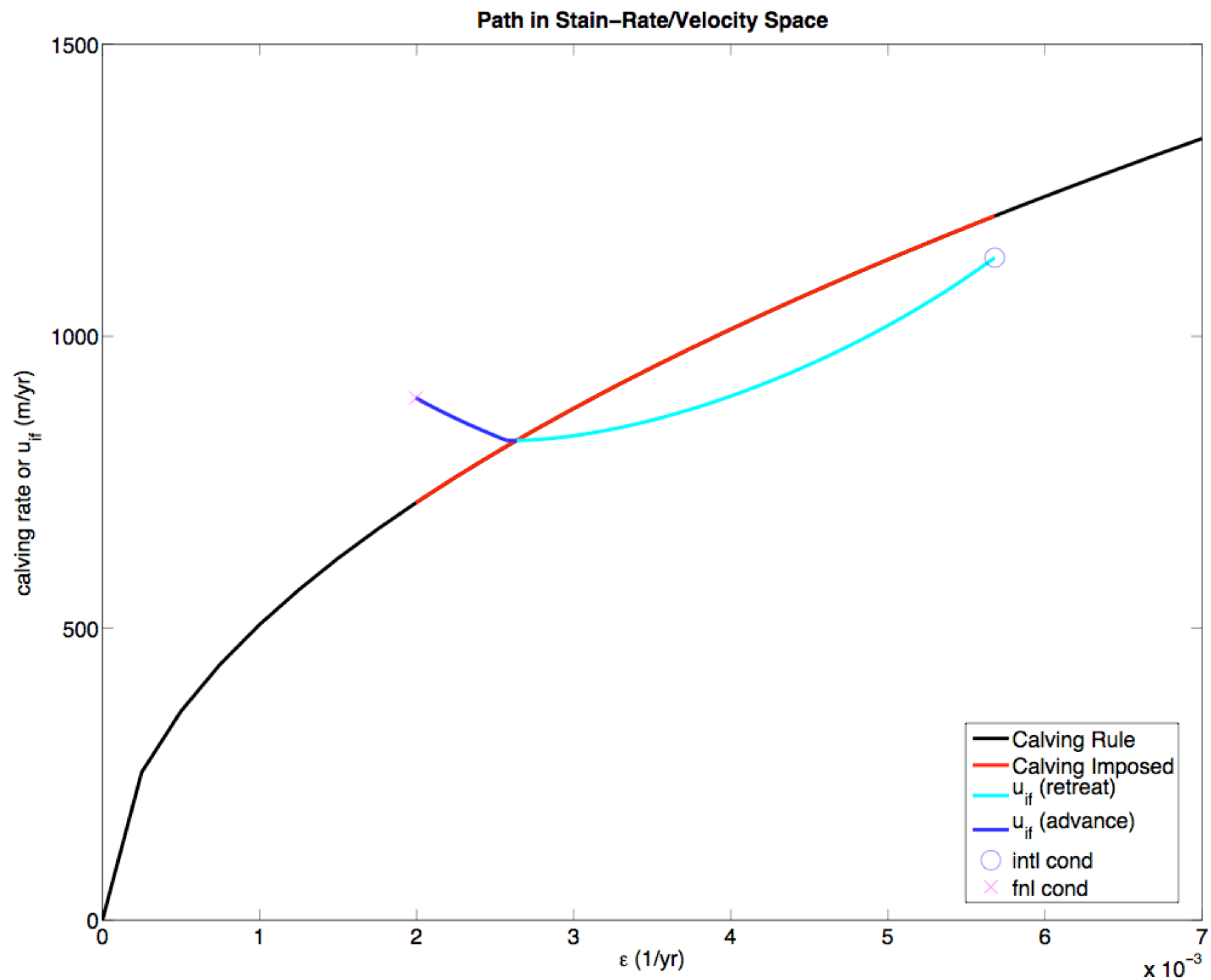
- Episodic version of the calving law
- Use strain-rate values an “berg width” back from the ice front to determine calving rate
- Variable width domain - data suggest that narrower shelves calve more slowly
- Ice islands - include local grounding



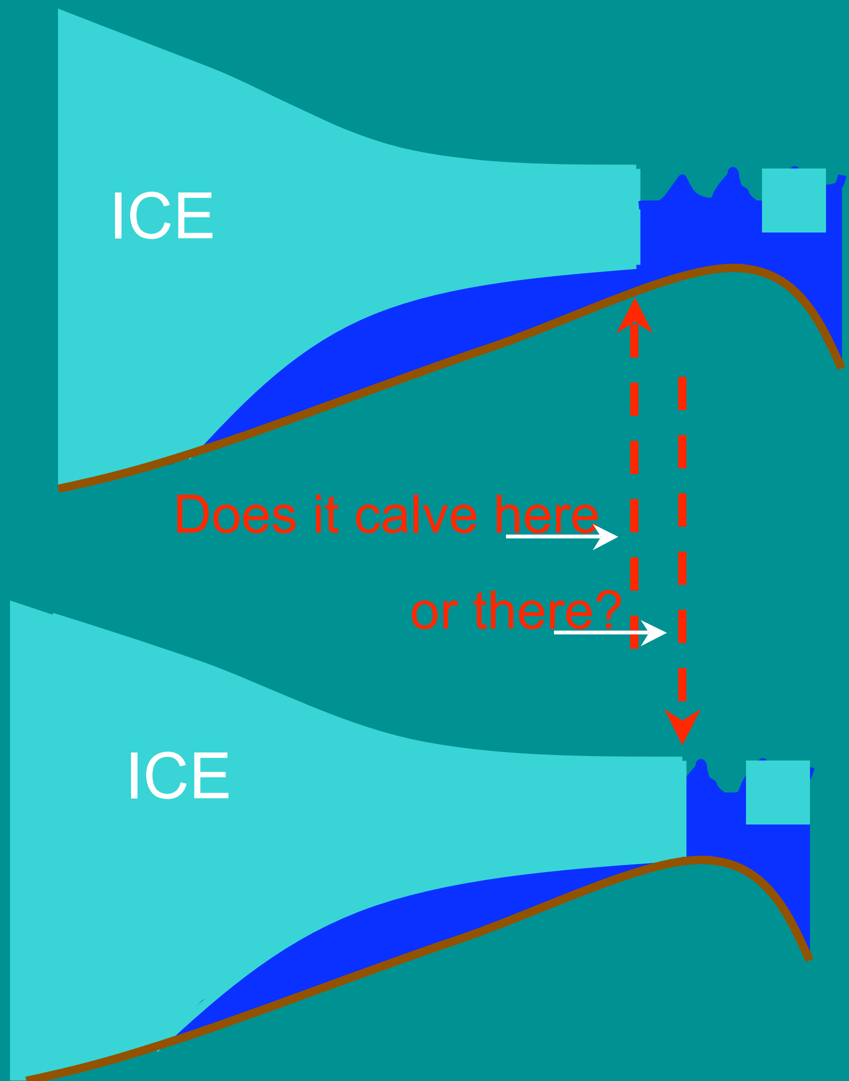
Surface and Bed Elev. Profiles
Elevation (m)
Initial
Final
Velocity Profiles
Velocity
Initial, Buttressed
Initial, Unbuttressed
Final
Flux Profiles
Depth–Int. Flux (m²/s)
Initial, Buttressed
Initial, Unbuttressed
Final
Along–flow Distance (km)



Ice Front Migration After Loss of 45% Buttressing
110
105
100
95
90
85
80
75
400
500
600
700
800
900
1000
1100
Time (yr)
x_if=Ice−Front Pos. (km)



Path in Stain–Rate/Velocity Space
calving rate or u_{if} (m/yr)
ε (1/yr)
x 10^{-3}
0
500
1000
1500
0
1
2
3
4
5
6
7
Calving Rule
Calving Imposed
u_{if} (retreat)
u_{if} (advance)
intl cond
fnl cond



Unbuttressed;
friction from local
high in bed not
stabilizing ice sheet.

Buttressed;
friction from local
high in bed is
stabilizing ice sheet.

To the best of our knowledge, no ice-sheet model calculates this physically. We should do better.

Very good reason why predicting hard for:

- Earthquakes;
- New volcanoes;
- Whether your ceramic coffee cup will break when you drop it on the floor;
- Iceberg calving from ice shelves...

Fracture mechanics is a mess! Depends on:

- Forcing (for ice shelves: tides, storms, collisions from passing icebergs, etc.);
- Material (temperature, c-axis fabric, impurities, etc.);
- Geometry (thickness, thickness gradient, transverse character, etc.);
- Pre-existing flaws (basal and surface crevasses, etc., hence history, such as tides at grounding line).

Engineers like to calculate everything to several decimal places, then build 3x stronger, just in case…

So, are we screwed? We hope not…

- Often, there is a "leading term" across a broad range of situations;
- A ceramic coffee cup over a hard floor will almost always survive a one-inch drop and almost always break from a ten-foot drop;
- Our hypothesis: tendency of ice shelves to fall apart (longitudinal stretching rate) controls the rate at which they fall apart (calving rate);
- Our hypothesis test: see if this explains a lot of the variance across a range of ice shelves.

There are lots of "issues"

- How steady is steady?
- Look how far back from the ice front?
- Smooth over what length of ice shelf?
- Etc.

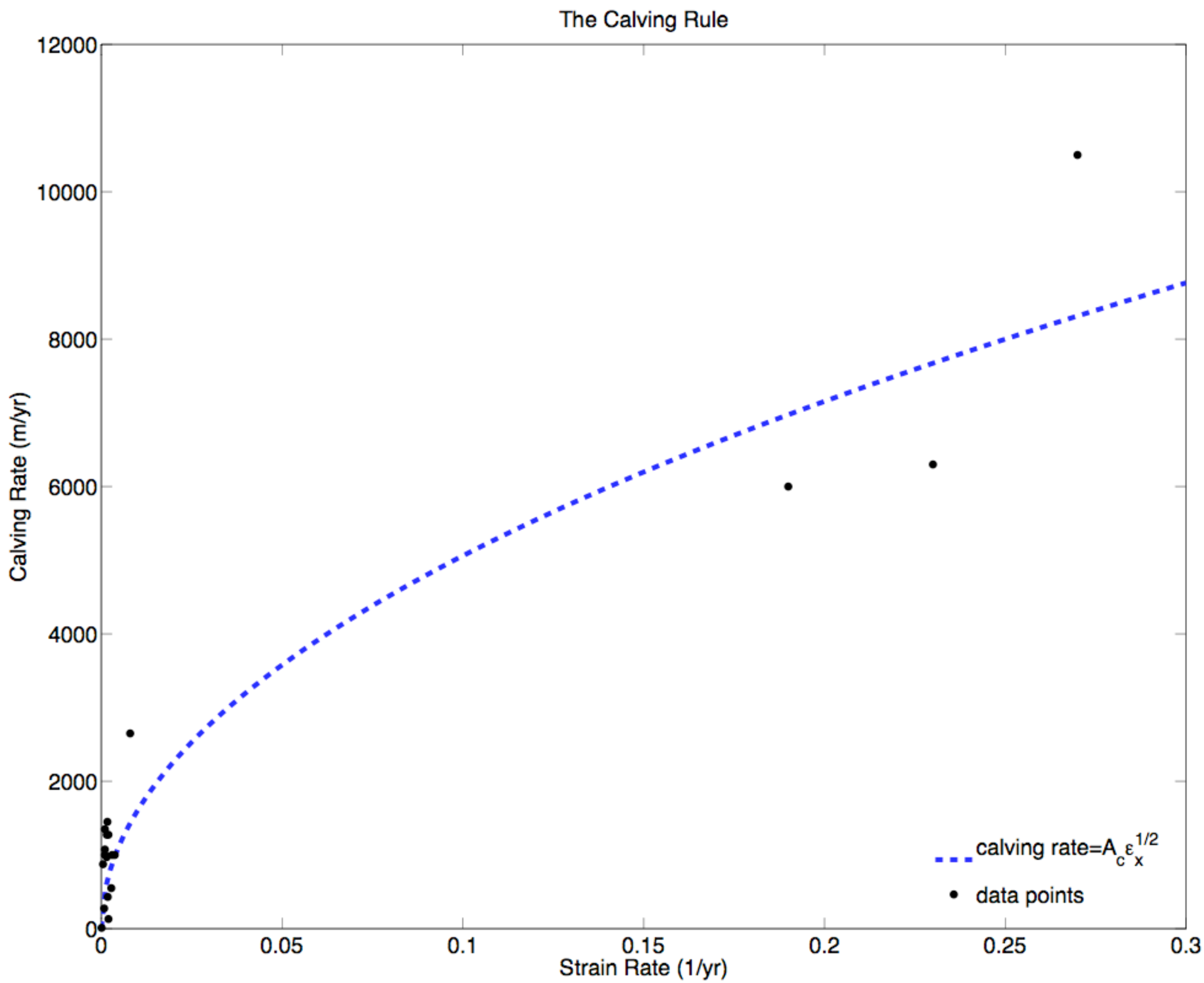
But, we are cautiously optimistic that these "issues" influence rather than control the outcome (I think that if you did the analysis, you'd get the same trend and similar numbers).

Calving rate increases with strain rate

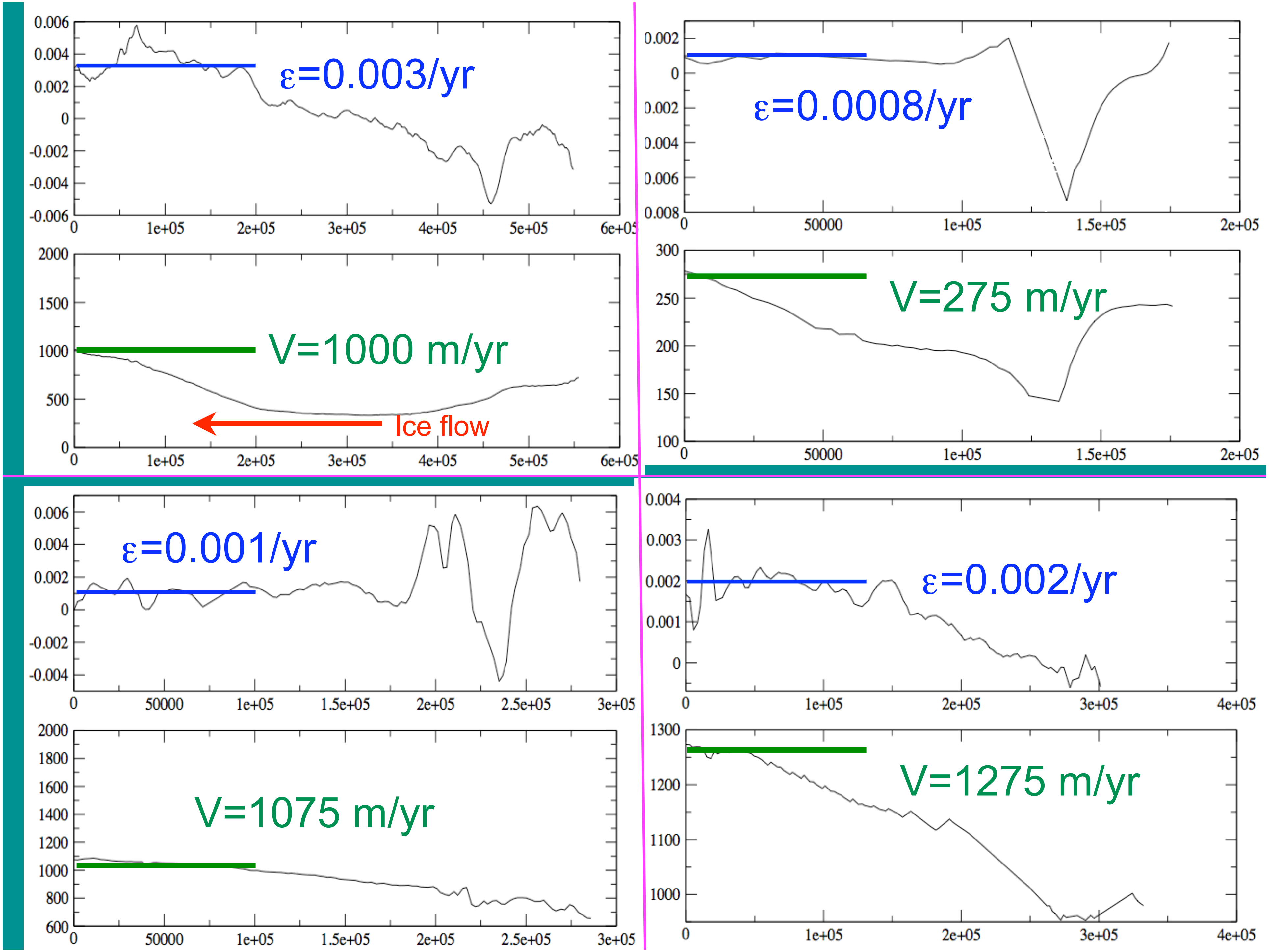
- We cannot yet find any obvious way out of this;
- This is what we hypothesized from common sense--ice shelves seem to fall apart in response to their tendency to fall apart;
- Regression gives calving rate $C \sim \varepsilon^k$, $k \sim 1/2$, ε=strain rate
- Tendency for basal crevasse of height L to grow under stretching-stress σ is measured by stress-intensity factor $K_{Ic} \sim L^{1/2}\sigma$. With $\varepsilon \sim \sigma^3$ and $L \sim \sigma$, $K_{Ic} \sim \varepsilon\, L^{1/2}$. Since $C \sim \varepsilon^{1/2}$, calving rate~ tendency for basal-crevasse growth? (This is probably too simplistic, but interesting…)

Synopsis

- Accurate modeling requires better calving laws, including one for cold ice shelves;
- An accurate, all-inclusive law won't be easy to find;
- We have gone looking for a first approximation;
- Our initial hypothesis--the calving rate increases with the longitudinal spreading rate (or, the rate at which the ice shelf falls apart increases with its tendency to fall apart) explains a lot of the variance across the ice shelves we have examined, with calving increasing as roughly the square root of spreading;
- Such a law may explain some ice-shelf behavior;
- We want more data, more-careful consideration of the physics, and more thought whether differences between shelves apply to single shelves;
- Pending those, we suggest $\text{calving\_rate} \sim \text{strain\_rate}^{1/2}$



The Calving Rule
12000
10000
8000
6000
4000
2000
0
0
0.05
0.1
0.15
0.2
0.25
0.3
Strain Rate (1/yr)
Calving Rate (m/yr)
calving rate=$A_c \varepsilon_x^{1/2}$
data points



ε=0.003/yr
V=1000 m/yr
Ice flow
ε=0.0008/yr
V=275 m/yr
ε=0.001/yr
V=1075 m/yr
ε=0.002/yr
V=1275 m/yr